

Exercise 31

For the following exercises, solve the equations over the complex numbers.

$$x^2 - 4x + 5 = 0$$

Solution

The two terms with x , x^2 and $4x$, cannot be combined, so it's necessary to complete the square to solve for x . Recall the following algebraic identity.

$$(x + B)^2 = x^2 + 2xB + B^2$$

Notice that $2B = -4$, which means $B = -2$ and $B^2 = 4$. Add and subtract 4 from the left side and apply the identity.

$$(x^2 - 4x + 4) + 5 - 4 = 0$$

$$(x + (-2))^2 + 1 = 0$$

$$(x - 2)^2 + 1 = 0$$

Now that x appears in only one place, it can be solved for. Subtract 1 from both sides.

$$(x - 2)^2 = -1$$

Take the square root of both sides.

$$\begin{aligned}\sqrt{(x - 2)^2} &= \sqrt{-1} \\ &= i\end{aligned}$$

Since there's an even power under an even root, and the result is to an odd power, an absolute value sign is needed around $x - 2$.

$$|x - 2| = i$$

Remove the absolute value sign by placing \pm on the right side.

$$x - 2 = \pm i$$

Add 2 to both sides.

$$x = 2 \pm i$$

Therefore, $x = \{2 - i, 2 + i\}$.